APPENDIX I
PROOF OF LEMMA II.1

(i) Consider the following set

$$\bar{\mathscr{P}} = \{\mathbf{p} \mid p_{ijk} \in \{0, \bar{p}_{ijk}\}, \mathbf{p} \in \mathscr{P}\}$$

and note that every feasible $\mathbf{p} \in \mathscr{P}$ can be expressed as a convex combination of the points in $\bar{\mathscr{P}}$. Thus Conv($\mathscr{P}$) is finitely generated by the points in $\bar{\mathscr{P}}$ and is therefore a polyhedron. Clearly, it is also bounded, hence a polytope (i.e., a bounded polyhedron). Since $\mathscr{R}$ is a linear mapping of $\mathscr{P}$, the same argument establishes that Conv($\mathscr{R}$) is a polytope.

(ii) Next, let $\mathbf{r} \in$ Conv($\mathscr{R}$) and suppose that the extreme points of Conv($\mathscr{R}$) are $\mathbf{r}^1, \ldots, \mathbf{r}^L$. These extreme points are also elements of $\mathscr{R}$, so they can be written as $\mathbf{r}^n = \mathbf{H}\mathbf{p}^n$ for some $\mathbf{p}^n \in \mathscr{P}$ and $n = 1, \ldots, L$. Thus, for $\alpha_n \geq 0$ satisfying $\sum_{n=1}^L \alpha_n = 1$,

$$\mathbf{r} = \sum_n \alpha_n \mathbf{r}^n = \sum_n \alpha_n \mathbf{H}\mathbf{p}^n = \mathbf{H}\mathbf{p},$$

where $\mathbf{p} = \sum_n \alpha_n \mathbf{p}^n \in$ Conv($\mathscr{P}$). Now, let $\mathbf{p} \in$ Conv($\mathscr{P}$) and let $\mathbf{p}^1, \ldots, \mathbf{p}^J$ be the extreme points of Conv($\mathscr{P}$). For $\beta_i \geq 0$ satisfying $\sum_i \beta_i = 1$, $\mathbf{p} = \sum_i \beta_i \mathbf{p}^i$. These extreme points are also elements of $\mathscr{P}$. Set $\mathbf{r}^i = \mathbf{H}\mathbf{p}^i$; we have $\mathbf{r}^i \in \mathscr{R}$. It follows

$$\mathbf{H}\mathbf{p} = \sum_i \beta_i \mathbf{H}\mathbf{p}^i = \sum_i \beta_i \mathbf{r}^i \in \text{Conv}(\mathscr{R}).$$

(iii) Let $\mathbf{r}$ be an extreme point of Conv($\mathscr{R}$). Then there exists a cost vector $\mathbf{c} \in \mathbb{R}^{(N+M)^2 K}$ such that $\mathbf{r}$ is the unique minimizer of $\mathbf{c}'\mathbf{x}$ over $\mathbf{x} \in$ Conv($\mathscr{R}$). Using part (ii), this latter problem is

$$\begin{array}{ll} \min & \mathbf{c}'\mathbf{x} \\ \text{s.t.} & \mathbf{x} = \mathbf{H}\mathbf{p}, \ \mathbf{p} \in \text{Conv}(\mathscr{P}). \end{array} \tag{28}$$

Let $(\mathbf{r}, \mathbf{p}^*)$ be an optimal solution. Then $\mathbf{p}^*$ is an optimal solution to

$$\begin{array}{ll} \min & \mathbf{c}'\mathbf{H}\mathbf{p} \\ \text{s.t.} & \mathbf{p} \in \text{Conv}(\mathscr{P}). \end{array} \tag{29}$$

(If not, there exists an optimal solution $\tilde{\mathbf{p}}$ of (29) and $\mathbf{H}\tilde{\mathbf{p}} \neq \mathbf{r}$ is optimal for (28), which contradicts the fact that $\mathbf{r}$ uniquely solves (28).) Now, there also exists some extreme point $\mathbf{p}^i$ of Conv($\mathscr{P}$) which is optimal for (29). In particular, $\mathbf{c}'\mathbf{H}\mathbf{p}^i = \mathbf{c}'\mathbf{H}\mathbf{p}^* = \mathbf{c}'\mathbf{r}$ and $\mathbf{H}\mathbf{p}^i$ also solves (28). Since $\mathbf{r}$ is a unique solution of (28) we have $\mathbf{r} = \mathbf{H}\mathbf{p}^i$.

APPENDIX II
THE DECOMPOSITION ALGORITHM FOR UTILITY MAXIMIZATION SUBJECT TO A LIFETIME CONSTRAINT

Suppose we have an extreme point $\mathbf{p}^1$ of Conv($\mathscr{P}$), which belongs to $\{\mathbf{p} \mid \mathbf{H}\mathbf{p} \in \mathscr{S}, \mathbf{C}\mathbf{p} \leq \boldsymbol{\chi}/T\}$ and let $m \in \{1, \ldots, J\}$, then the restricted master problem at the mth iteration is

$$\begin{array}{ll} \min & -F(\mathbf{H}\mathbf{p}) \\ \text{s.t.} & \mathbf{p} - \sum_{n=1}^m \alpha_n \mathbf{p}^n = 0, \\ & \sum_{n=1}^m \alpha_n = 1, \\ & \sum_{j=1}^{N+M} \frac{p_{ijk}G_{ij} - p_{jik}G_{ji}}{\eta \ln 2} = 0, \ \forall i \neq s(k), d(k), \forall k, \\ & \mathbf{A}\mathbf{H}\mathbf{p} \leq \mathbf{b}, \\ & \mathbf{C}\mathbf{p} \leq \boldsymbol{\chi}/T, \\ & \alpha_n \geq 0, \ n = 1, \ldots, m, \end{array} \tag{30}$$

and the corresponding subproblem is

$$\begin{array}{ll} \max & \boldsymbol{\lambda}'\mathbf{p} \\ \text{s.t.} & \mathbf{p} \in \mathscr{P}, \end{array} \tag{31}$$

with cost vector $\boldsymbol{\lambda} = \boldsymbol{\lambda}^{(m)}$, where as before we have $(\mathbf{p}^{(m)}, \boldsymbol{\alpha}^{(m)}; \boldsymbol{\lambda}^{(m)}, \mu^{(m)}, \boldsymbol{\nu}^{(m)}, \boldsymbol{\sigma}^{(m)}, \boldsymbol{\xi}^{(m)})$ as the optimal primal-dual pair for the restricted master problem (30) with $\boldsymbol{\xi}^{(m)}$ being the optimal dual variable corresponding to the lifetime constraint.

Following the same recipe as described in Fig. 1, we can obtain an optimal solution to problem (24) in a finite number of iterations. The argument is almost identical and is therefore omitted for brevity. Furthermore, problem (31) is still equivalent to the maximum weighted matching problem constructed in a similar way as in Section V, and is solvable in polynomial time. In particular, to solve problem (31), we construct the same undirected graph $\mathscr{G} = (\mathscr{V}, \mathscr{E})$. Let $\delta_{ijk} = \lambda_{ijk}\bar{p}_{ijk}$ $\forall i, j, k$, and the weight for each edge $(i, j) \in \mathscr{E}$ is given by

$$w_{ij} = \begin{cases} \max\limits_{k=1,\ldots,K} \max\{\delta_{ijk}, \delta_{jik}, 0\}, & \forall i, j \in \mathscr{A}, \\ \max\limits_{k=1,\ldots,K} \max\{\delta_{i,N+l,k}, 0\}, & \forall i \in \mathscr{A}, j \in \mathscr{B}_l, \\ \max\limits_{k=1,\ldots,K} \max\{\delta_{j,N+l,k}, 0\}, & \forall i \in \mathscr{B}_l, j \in \mathscr{A}, \\ 0 & \text{otherwise}. \end{cases}$$

Furthermore, we construct the set $\mathscr{K}$ as follows. For each $1 \leq i \leq N$, $1 \leq j \leq N + M$, we select only one, if any, k satisfying the conditions

$$k = \begin{cases} \text{argmax}_{t=1,\ldots,K} \max\{\delta_{ijt}, \delta_{jit}, 0\}, & \text{if } j \leq N, \\ \text{argmax}_{t=1,\ldots,K} \max\{\delta_{ijt}, 0\}, & \text{otherwise}, \end{cases}$$

and

$$\begin{cases} \delta_{ijk} = \max\{\delta_{ijk}, \delta_{jik}, 0\} > 0, & \text{if } j \leq N, \\ \delta_{ijk} > 0, & \text{otherwise}, \end{cases}$$

and let (i, j, k) be an element of $\mathscr{K}$. Given the graph $\mathscr{G}$ constructed above, we can obtain the optimal solution of (31) in the same way as in Section V.