

# Problem Set 1

Due: Monday, February 4, 2008

**Note:** We will refer to the text as [YG]. Starred problems are harder than the rest.

**Readings:** Course Notes for Lectures 0–2 and [YG] Chapter 1 (Sections 1.1–1.4).

## Problem 1

Show the following statements:

- (a)  $(A \cap B)^c = A^c \cup B^c$ .
- (b) Starting from the commutative law for the union operation show  $C \cap D = D \cap C$ .
- (c) Let  $S$  be the universal set. For any set  $A$  we have that  $S = A \cup A^c$ . Show that  $S^c = \emptyset$ .

## Problem 2: [YG] Problem 1.2.1

## Problem 3

During a 24h period a computer is to be accessed at time  $X$ , used for some processing, and exited at time  $Y \geq X$ . Take  $X$  and  $Y$  to be measured in hours on the time line with the beginning of the 24h period as the origin. The experiment is to observe the two-dimensional random vector  $(X, Y)$ .

- (a) Describe the sample space.
- (b) Sketch the following events in the  $X, Y$  plane:
  - (i) The time of use is 1h or less.
  - (ii) The access is before  $t_1$  and the exit after  $t_2$ , where  $0 \leq t_1 < t_2 \leq 24$ .
  - (iii) The time of use is less than 20% of the 24h period.

## Problem 4: [YG] Problem 1.2.3

## Problem 5: [YG] Problem 1.3.1

## Problem 6: [YG] Problem 1.4.2

## Problem 7

- (a) Show  $\mathbf{P}[A \cup B] = \mathbf{P}[A] + \mathbf{P}[B] - \mathbf{P}[A \cap B]$ .

\*(b) Show that

$$\begin{aligned} \mathbf{P}[\cup_{i=1}^k A_i] &= \sum_{i=1}^k \mathbf{P}[A_i] - \sum_{i < j=2}^k \mathbf{P}[A_i \cap A_j] + \sum_{i < j < r=3}^k \mathbf{P}[A_i \cap A_j \cap A_r] \\ &\quad + \cdots + (-1)^{k-1} \mathbf{P}[A_1 \cap A_2 \cap \cdots \cap A_k] \end{aligned}$$

**Problem 8:** [YG] **Problem 1.4.5**

**Problem 9**

Show that for any two not necessarily disjoint events  $A, B$  the probability that one and only one of the events  $A$  or  $B$  occurs is:

$$P[A] + P[B] - 2 \cdot P[A \cap B].$$

**Problem 10**

- (a) Let  $(X, Y)$  denote the genre of children in a family with two children. What is the sample space for  $(X, Y)$  ?
- (b) Assume that boys and girls are equally likely. Bob is one of two children in **his** family. What is the probability that the *other child* is Bob's sister.