Problem Set 3

Due: Wednesday, February 20, 2008

Note: We will refer to the text as [YG]. Starred problems are harder than the rest.

Readings: Course Notes for Lectures 5–6 and [YG] Chapter 2.

Problem 1: [YG] Problem 2.2.5

Problem 2

Each morning, Hungry Harry eats some eggs. On any given morning, the number of eggs he eats is equally likely to be 1,2,3,4,5 or 6, independently of what he has done in the past. Let X be the number of eggs Harry eats in 10 days. Find:

(a) $\mathbf{E}[X]$.

(b) $\mathbf{V}[X]$.

Problem 3

An internet access provider (IAP) owns two servers. Each server has a 50% chance of being "down" independently of the other. Fortunately, only one server is necessary to allow the IAP to provide service to its customers,i.e., only one server is needed to keep the IAP's system up. Suppose a customer tries to access the internet on four different occasions, and assume that the state of the system at any time is independent of the state of the system at any other time. What is the probability that the customer will only be able to access the internet on 3 out of the 4 calls ?

Problem *4

For $X_1, \dots X_n$ positive, independent and identically distributed (iid), random variables find

$$\mathbf{E}\left[\frac{\sum_{i=1}^{m} X_i}{\sum_{i=1}^{n} X_i}\right]$$

for $m \leq n$.

Problem 5

A receiver receives signals sent by a particular transmitter. There is a $\frac{1}{6}$ probability that the signal received will be a 1, a $\frac{1}{6}$ probability that it will be a 0, and a $\frac{2}{3}$ probability that the signal will be unintelligible. Define the random variable X to be the number of signals (intelligible or not) received up to and including the first 1. Define Y to be the number of signals received up to and including the first 0.

(a) Find $E[X]$.

(b) Find $E[X|Y = 1]$.

Problem 6: [YG] Problem 2.3.5

Problem 7

The following is known as the St. Petersburg paradox. Imagine that someone proposes to play the following game with you: you flip a coin, until the first tail appears. If the tail appears on the nth flip of the coin, you receive 2^n dollars. What is your expected gain?

Problem 8: [YG] Problem 2.8.10

Problem 9: [YG] Problem 2.9.7

Problem 10: [YG] Problem 2.10.6