

## Problem Set 4

Due: Wednesday, February 27, 2008

**Note:** We will refer to the text as [YG]. Starred problems are harder than the rest.

**Readings:** Course Notes for Lectures 7–9 and [YG] Chapter 3.

### Problem 1

Consider an r.v.  $X$  with density function

$$f_X(x) = \begin{cases} kx & \text{if } 0 \leq x < 2 \\ k(4-x) & \text{if } 2 \leq x \leq 4 \\ 0 & \text{otherwise} \end{cases}$$

- (a) Find the value of  $k$  for which  $f_X(x)$  is a legitimate density function.
- (b) Find the mean and variance of  $X$ .
- (c) Find the cumulative distribution function (CDF) of  $X$ .
- (d) Find  $\mathbf{P}[1 < X \leq 3]$ .

**Problem 2:** [YG] Problem 3.2.5

**Problem 3:** [YG] Problem 3.3.8

**Problem 4:** [YG] Problem 3.4.9

**Problem 5:** [YG] Problem 3.5.3

**Problem 6:** [YG] Problem 3.5.6

### Problem 7

Consider a random variable  $X$  with density

$$f_X(x) = \begin{cases} ax & 0 \leq x \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

- (a) What's the value of  $a$  for  $f_X(x)$  to be a legitimate density function ?

(b) Let  $Y = \sqrt{X}$ . Find first the distribution and then the density of  $Y$ .

**Problem 8**

Let  $X$  be a random variable with probability density function  $f_X(x)$ . Define a new random variable  $Y = |X|$ . Find the density for  $Y$ .

**Problem 9: [YG] Problem 3.8.7**