Lecture 1: Outline

- Administrative stuff.
- Some History.
- LP flavors.
History of Optimization

Fermat, 1638; Newton, 1670
\[ \min f(x) \quad x: \text{scalar} \]
\[ \frac{df(x)}{dx} = 0 \]

Euler, 1755
\[ \min f(x_1, \ldots, x_n) \]
\[ \nabla f(x) = 0 \]

Lagrange, 1797
\[ \min f(x_1, \ldots, x_n) \]
\[ \text{s.t. } g_k(x_1, \ldots, x_n) = 0, \quad k = 1, \ldots, m. \]

Euler, Lagrange Problems in infinite dimensions \((n \to \infty)\), calculus of variations.

Linear Programming (LP)

minimize \[ 3x_1 + x_2 \]
\[ \text{s.t. } x_1 + 2x_2 \geq 2 \]
\[ 2x_1 + x_2 \geq 2 \]
\[ x_1 \geq 0 \]
\[ x_2 \geq 0 \]

in vector notation
\[ c = \begin{bmatrix} 3 \\ 1 \end{bmatrix} \quad x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \quad b = \begin{bmatrix} 2 \\ 2 \end{bmatrix} \quad A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} \]
\[ \text{minimize } c'x \]
\[ \text{s.t. } Ax \geq b \]
\[ x \geq 0 \]
LP History

George Dantzig, 1947  Simplex method.
Fourier, 1826  Method for solving system of linear inequalities.
de la Vallée Poussin  simplex-like method for objective function with absolute values.
Kantorovich, Koopmans, 1930s  Formulations and solution method.
von Neumann, 1928  game theory, duality.
Farkas, Minkowski, Carathéodory, 1870-1930  Foundations.

  1950s  Applications.
  1960s  Large Scale Optimization.
  1970s  Complexity theory.
Khachyan, 1979  The ellipsoid algorithm.
Karmakar, 1984  Interior point algorithms.

Applications of LP

- Transportation (WW II, air traffic control, crew scheduling, etc.)
- Telecommunications (routing, scheduling, resource allocation)
- Manufacturing (production planning, scheduling, resource allocation)
- Medicine, Computational Biology (metabolic networks, protein side-chain packing).
- Engineering.
- Typesetting (\TeX, \LaTeX)
Possible solution outcomes

1. There exists a **unique optimal solution**.
2. There exist **multiple optimal solutions** (their set being either bounded or unbounded).
3. Optimal cost is $-\infty$ and no feasible solution is optimal (**unbounded problem**).
4. Feasible set is empty (**infeasible problem**).

Various LP Flavors

**General LP**

\[
\begin{align*}
\text{minimize} & \quad c^T x \\
\text{s.t.} & \quad A x \geq a \\
& \quad B x \leq b \\
& \quad D x = d \\
& \quad x_i \geq 0, \quad i \in I \\
& \quad x_j \leq 0, \quad j \in J.
\end{align*}
\]

reduces to

\[
\begin{align*}
\text{minimize} & \quad c^T x \\
\text{s.t.} & \quad A x \geq b
\end{align*}
\]

**Standard form LP**

\[
\begin{align*}
\text{minimize} & \quad c^T x \\
\text{s.t.} & \quad A x = b \\
& \quad x \geq 0
\end{align*}
\]

**Remark**

Every LP can be written in standard form.