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- Administrative stuff.
- Some History.
- LP flavors.

History of Optimization

Fermat, 1638; Newton, 1670

$$\min f(x) \quad x: \text{scalar}$$

$$\frac{df(x)}{dx} = 0$$

Euler, 1755

$$\min f(x_1, \dots, x_n)$$

$$\nabla f(\mathbf{x}) = 0$$

Lagrange, 1797

$$\min f(x_1, \dots, x_n)$$

$$\text{s.t. } g_k(x_1, \dots, x_n) = 0, \quad k = 1, \dots, m.$$

Euler, Lagrange Problems in infinite dimensions ($n \rightarrow \infty$), calculus of variations.

Linear Programming (LP)

$$\begin{aligned} &\text{minimize} && 3x_1 + x_2 \\ &\text{s.t.} && x_1 + 2x_2 \geq 2 \\ &&& 2x_1 + x_2 \geq 2 \\ &&& x_1 \geq 0 \\ &&& x_2 \geq 0 \end{aligned}$$

in **vector notation**

$$\mathbf{c} = \begin{bmatrix} 3 \\ 1 \end{bmatrix} \quad \mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \quad \mathbf{b} = \begin{bmatrix} 2 \\ 2 \end{bmatrix} \quad \mathbf{A} = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$$

$$\begin{aligned} &\text{minimize} && \mathbf{c}'\mathbf{x} \\ &\text{s.t.} && \mathbf{A}\mathbf{x} \geq \mathbf{b} \\ &&& \mathbf{x} \geq 0 \end{aligned}$$

LP History

George Dantzig, 1947 Simplex method.

Fourier, 1826 Method for solving system of linear inequalities.

de la Vallée Poussin simplex-like method for objective function with absolute values.

Kantorovich, Koopmans, 1930s Formulations and solution method.

von Neumann, 1928 game theory, duality.

Farkas, Minkowski, Carathéodory, 1870-1930 Foundations.

1950s Applications.

1960s Large Scale Optimization.

1970s Complexity theory.

Khachyan, 1979 The ellipsoid algorithm.

Karmakar, 1984 Interior point algorithms.

Applications of LP

- Transportation (WW II, air traffic control, crew scheduling, etc.)
- Telecommunications (routing, scheduling, resource allocation)
- Manufacturing (production planning, scheduling, resource allocation)
- Medicine, Computational Biology (metabolic networks, protein side-chain packing).
- Engineering.
- Typesetting (T_EX, L^AT_EX)

Possible solution outcomes

- 1 There exists a **unique optimal solution**.
- 2 There exist **multiple optimal solutions** (their set being either bounded or unbounded).
- 3 Optimal cost is $-\infty$ and no feasible solution is optimal (**unbounded problem**).
- 4 Feasible set is empty (**infeasible problem**).

Various LP Flavors

$$\begin{aligned}
 \text{(General LP)} \quad & \text{minimize} \quad c^T x \\
 & \text{s.t.} \quad Ax \geq a \\
 & \quad \quad Bx \leq b \\
 & \quad \quad Dx = d \\
 & \quad \quad x_i \geq 0, \quad i \in I \\
 & \quad \quad x_j \leq 0, \quad j \in J.
 \end{aligned}$$

$$\begin{aligned}
 \text{reduces to} \quad & \text{minimize} \quad c^T x \\
 & \text{s.t.} \quad Ax \geq b
 \end{aligned}$$

$$\begin{aligned}
 \text{(Standard form LP)} \quad & \text{minimize} \quad c^T x \\
 & \text{s.t.} \quad Ax = b \\
 & \quad \quad x \geq 0
 \end{aligned}$$

Remark

Every LP can be written in standard form.