

SE524/EC524 Optimization Theory and Methods

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Case Study



Lecture 12: Outline

- 1 Parametric Programming.
- 2 The V. Sattui winery case study.
- 3 Some sample problems.

Inventory planning: V. Sattui winery case study

The Vittorio Sattui (V. Sattui) winery has contracted to supply during the next 4 years the following units of its Morisoli Vineyard Cabernet to the Mistral restaurant:

Year	2000	2001	2002	2003
Units	150	160	225	180

- The Morisoli vineyard can produce up to 160 units of Cabernet per year at a cost of \$35 per unit.
- Additional units can be substituted by an Italian Barolo at a cost of \$50 per unit.
- Inventory holding cost: \$5 per year for each unit of wine.

Objective

Find an inventory plan (what to produce, store, order) to minimize the contract execution cost

Inventory planning: Formulation

- x_i : # of units of wine produced during years $i = 1, 2, 3, 4$.
- s_i : # of units of Barolo ordered during year i .
- l_i : inventory (in units of wine) held at the end of year i .

minimize $35 \sum_{i=1}^4 x_i + 50 \sum_{i=1}^4 s_i + 5 \sum_{i=1}^4 l_i$
 subject to

$$c1 : x_1 + s_1 - l_1 = 150,$$

$$c2 : x_2 + s_2 + l_1 - l_2 = 160,$$

$$c3 : x_3 + s_3 + l_2 - l_3 = 225,$$

$$c4 : x_4 + s_4 + l_3 - l_4 = 180,$$

$$c5 : x_1 \leq 160,$$

$$c6 : x_2 \leq 160,$$

$$c7 : x_3 \leq 160,$$

$$c8 : x_4 \leq 160,$$

$$x_i, s_i, l_i \geq 0, \quad \forall i.$$

Inventory planning: Solver output

Primal optimal solution:

x_1^*	x_2^*	x_3^*	x_4^*	s_1^*	s_2^*	s_3^*	s_4^*	l_1^*	l_2^*	l_3^*	l_4^*
160	160	160	160	0	0	55	20	10	10	0	0

Sensitivity analysis:

OBJ Sensitivity Ranges				
Variable Name	Reduced Cost	Down	Current	Up
x_1	zero	-infinity	35.0000	40.0000
x_2	zero	-infinity	35.0000	45.0000
x_3	zero	-infinity	35.0000	50.0000
x_4	zero	-infinity	35.0000	50.0000
s_1	10.0000	40.0000	50.0000	+infinity
s_2	5.0000	45.0000	50.0000	+infinity
s_3	zero	45.0000	50.0000	55.0000
s_4	zero	35.0000	50.0000	55.0000
l_1	zero	-5.0000	5.0000	10.0000
l_2	zero	zero	5.0000	10.0000
l_3	5.0000	zero	5.0000	+infinity
l_4	55.0000	-50.0000	5.0000	+infinity

Inventory planning: Solver output (cont.)

RHS Sensitivity Ranges				
Constraint Name	Dual Price	Down	Current	Up
c1	40.0000	95.0000	150.0000	160.0000
c2	45.0000	105.0000	160.0000	170.0000
c3	50.0000	170.0000	225.0000	+infinity
c4	50.0000	160.0000	180.0000	+infinity
c5	-5.0000	150.0000	160.0000	215.0000
c6	-10.0000	150.0000	160.0000	215.0000
c7	-15.0000	zero	160.0000	215.0000
c8	-15.0000	zero	160.0000	180.0000

Inventory planning: Questions

(Q1) The winery is considering preventive maintenance during one of the first 3 years. If maintenance is scheduled:

- in 2000, capacity becomes 151 units (instead of 160).
- in 2001, capacity becomes 153 units (instead of 160).
- in 2002, capacity becomes 155 units (instead of 160). **When do you recommend that maintenance takes place? Why?**

First note that the changes in \mathbf{b} are within the allowable range.

$$F(\mathbf{b}) = F(\mathbf{b}^*) + \mathbf{p}'(\mathbf{b} - \mathbf{b}^*).$$

Cost difference in

- 2000: $-5 \times -9 = 45$.
- 2001: $-10 \times -7 = 70$.
- 2002: $-15 \times -5 = 75$.

⇒ **Schedule maintenance in 2000.**

Inventory planning: Questions (cont.)

(Q2) Another winery has offered to supply up to 50 units of Cabernet during either 2000, 2001, or 2002 at a price of \$45 per unit. **Should the V. Sattui winery buy from this other winery? If yes, when and how many units? What is the impact of this decision on the total cost?**

By looking at the optimal solution,

x_1^*	x_2^*	x_3^*	x_4^*	s_1^*	s_2^*	s_3^*	s_4^*	I_1^*	I_2^*	I_3^*	I_4^*
160	160	160	160	0	0	55	20	10	10	0	0

it makes sense to buy only during 2002. In particular, V. Sattui can buy 50 units and reduce s_3^* to 5 units, realizing savings of $\$5 \times 50 = \250 .

Inventory planning: Questions (cont.)

(Q3) The provider of the Italian Barolo wine has offered to lower the price (from \$50 per unit) during 2001. **What is the minimum decrease that would make this offer attractive to the V. Sattui winery?**

Note that $s_2^* = 0$, so V. Sattui is not using Barolo during 2001. To increase s_2^* from zero the cost is \$5 per unit (the reduced cost of variable s_2). So the price of the Barolo should be decreased by at least \$5 per unit to make it attractive during 2001.

Inventory planning: Questions (cont.)

(Q4) Because of anticipated inflation and increases in interest rates, the inventory holding cost is expected to increase to \$8 per unit during 2001. **How does this affect the total cost and the optimal solution?**

The cost of l_2 can be increased up to \$10 without affecting the optimal basis. Thus, the increase to \$8 is within this range. The optimal solution will not change but the cost will increase by $\$3 \times 10 = \30 since $l_2^* = 10$.