SE524/EC524 Optimization Theory and Methods

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Lecture 23: Outline

1. Practical Guidelines for solving LPs.
2. Case study: The fleet assignement problem.

Practical Guidelines for LPs

- Exploit sparsity.
- Start with a "good" basis in simplex.
- Use of dual simplex.
- Use of interior point methods. Simplex or interior point method ?

Fleet assignement

Given fleet and flight schedule **assign aircrafts to flights**.

Fleet information

- $\mathscr{F}$: set of fleets.
- $S(f)$: # of aircraft in fleet $f \in \mathscr{F}$.

Fleet assignement (cont.)

Schedule information

- $\mathscr{C}$: set of cities served
- $\mathscr{L}$: set of flights
- $(o, d, t) \in \mathscr{L}$: (origin, dest., dep. time)
- c_{fodt}: cost
- t_0: reference time
- $(t_1, \ldots, t_n)$: time partition
- t^-, t^+
- $t(f, o, d)$: time that fleet f does o–d
- $O(t_0)$: set of flights on the air in $[t_0, t_0^+]$
- $\mathscr{H}$: set of "required through" pairs of flights (to be served by same fleet)

Formulation

Objective: Minimize cost

Decision variables

$$x_{fodt} = \begin{cases} 1 & \text{if fleet f for } (o, d, t) \\ 0 & \text{otherwise} \end{cases}$$

$$y_{fot} = \# \text{ of aircraft from } f \text{ on ground in } o \text{ during } [t, t^+]$$

$$z_{fot} = \# \text{ of aircraft from } f \text{ that arrive in } o \text{ at } t$$

Formulation (cont.)

$$\min \sum_{f\in\mathscr{F}} \sum_{(o,d,t)\in\mathscr{L}} c_{fodt} x_{fodt}$$

$$\sum_{f\in\mathscr{F}} x_{fodt} = 1 \quad \forall (o,d,t) \in \mathscr{L}$$

$$z_{fot} + y_{fot^-} - \sum_{d\in\mathscr{C}} x_{fodt} - y_{fot} = 0 \quad \forall f, o, t$$

$$x_{fodt} - x_{fdd't'} = 0 \quad \forall f \in \mathscr{F}, ((o,d,t),(d,d',t')) \in \mathscr{H}$$

$$\sum_{(o,d,t)\in O(t_0)} x_{fodt} + \sum_{o\in\mathscr{C}} y_{fot_0} \le S(f) \quad \forall f \in \mathscr{F}$$

$$z_{fot} = \sum_{\{(d,o,\tau)\in\mathscr{L} \mid \tau + t(f,d,o)=t\}} x_{fdo\tau}$$

$$x_{fodt} \in \{0,1\},\ y_{fot} \ge 0,\ y_{fot} : \text{integer}$$

Algorithm

1. Preprocessing.
2. Perturb all costs.
3. Dual simplex or path following on LP relaxation.
4. Remove perturbation.
5. Reoptimize with original cost.
6. Round variables above 0.99 to 1.
7. Preprocessing.
8. Dual simplex.
9. Branch and bound.