SE524/EC524 Optimization Theory and Methods

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Lecture 7: Outline

1. Revised Simplex method.
2. Cycling and how to avoid it.
3. Finding an initial bfs: Phase I.
4. Possible outcomes of the simplex method.
5. On the efficiency of the simplex method.

Lexicographic pivoting rule

Recall that at a degenerate bfs we might not move but only change basis ($\theta^* = 0$). Unfortunately, it is possible to change a series of basis and return to the same one. Then the algorithm never terminates.

Definition

$\mathbf{u} \in \mathbb{R}^n$ is said to be **lexicographically larger (or smaller)** than $\mathbf{v} \in \mathbb{R}^n$ if $\mathbf{u} \neq \mathbf{v}$ and the first $\neq 0$ component of $\mathbf{u} - \mathbf{v}$ is > 0 (or < 0, respectively). We write $\mathbf{u} \overset{L}{>} \mathbf{v}$ ($\mathbf{u} \overset{L}{<} \mathbf{v}$, respectively).

Lexicographic pivoting rule

1. Pick arbitrary column $\mathbf{A}_j$ with $\bar{c}_j < 0$.
2. $\forall i$ with $x_{ij} > 0$ divide ith row by x_{ij} and choose as pivot row the lexicographically smallest.

Lexicographic pivoting rule (cont.)

Theorem

Suppose that simplex starts with all rows $\overset{L}{>} 0$ (except 0th) and the lexicographic pivoting rule is followed. Then

a. All rows (except 0th) remain $\overset{L}{>} 0$ throughout the algorithm.
b. 0th row strictly increases lexicographically at every iteration.
c. Simplex terminates in a finite number of iterations.

Bland's anticycling rule

1. Pivot column: Smallest j with $\bar{c}_j < 0$.
2. Pivot row: Break possible tie by picking the row with smallest i.

Finding an initial bfs: Phase I

1. Multiply rows with -1 to get $\mathbf{b} \geq \mathbf{0}$.
2. Introduce artificial variables $\mathbf{y}$ and apply simplex to auxiliary problem

$$\min \sum_{i=1}^{m} y_i$$
$$s.t.\ \mathbf{A}\mathbf{x} + \mathbf{y} = \mathbf{b}, \quad \mathbf{x}, \mathbf{y} \geq \mathbf{0}$$

3. If cost > 0 $\Rightarrow$ **infeasible**. STOP.
4. If cost $= 0$ but no artificial variable is in the basis then we have a bfs of the original problem.
5. Else, **drive artificial variables out of the basis**: If lth basic variable is artificial examine lth row of $\mathbf{B}^{-1}\mathbf{A}$. If all elements $= 0 \Rightarrow$ row redundant. Otherwise pivot with $\neq 0$ element.

Possible outcomes of the simplex method

1. **Infeasible**: Detected at Phase I.
2. **A has lin. dependent rows**: Detected at Phase I, eliminate redundant rows.
3. **Unbounded** (cost$= -\infty$): detected at Phase II.
4. **Optimal solution**: Terminate at Phase II in optimality check.