Inventory Planning Problem

The Vittorio Sattui (V. Sattui) winery of Napa Valey has agreed to supply during the next 4 years the following units of Cabernet Sauvignon wine from its Morisoli Vineyard to the Mistral restaurant in Boston:

Year	2000	2001	2002	2003
Units	150	160	225	180

The Morisoli vineyard can produce a maximum of 160 units of Cabernet per year at a cost of \$35 per unit. Additional units can be substituted by a fine Italian Barolo at a cost of \$50 per unit. The V. Sattui winery incurs an inventory holding cost of \$5 per year for each unit of wine.

(a) Let x_i denote the number of units of wine produced at the Morisoli vineyard during year $i = 1, 2, 3, 4$ (2000 through 2003). Let s_i the number of units of the Italian Barolo wine used to satisfy the contract with the Mistral restaurant during year $i = 1, 2, 3, 4$. Let finally I_i the inventory (in units of wine) held at the winery at the end of year $i = 1, 2, 3, 4$ (the inventory at the end of 1999 is zero). Obviously, the V. Sattui winery wants to minimize its cost of honoring the contract with Mistral. Formulate this problem as a linear programming problem in standard form.

Solving the problem (you formulated above) with an LP solver you obtain optimal cost equal to \$26,250, optimal primal solution given by

x_1^*	x_2^*	x_3^*	x_4^*	s_1^*	s_2^*	s_3^*	s_4^*	I_1^*	I_2^*	I_3^*	I_4^*
160	160	160	160	0	0	55	20	10	10	0	0

and the following sensitivity analysis information

OBJ Sensitivity Ranges				
Variable Name	Reduced Cost	Down	Current	Up
x_1	zero	-infinity	35.0000	40.0000
x_2	zero	-infinity	35.0000	45.0000
x_3	zero	-infinity	35.0000	50.0000
x_4	zero	-infinity	35.0000	50.0000
s_1	10.0000	40.0000	50.0000	+infinity
s_2	5.0000	45.0000	50.0000	+infinity
s_3	zero	45.0000	50.0000	55.0000
s_4	zero	35.0000	50.0000	55.0000
I_1	zero	-5.0000	5.0000	10.0000
I_2	zero	zero	5.0000	10.0000
I_3	5.0000	zero	5.0000	+infinity
I_4	55.0000	-50.0000	5.0000	+infinity

RHS Sensitivity Ranges				
Constraint Name	Dual Price	Down	Current	Up
c1	40.0000	95.0000	150.0000	160.0000
c2	45.0000	105.0000	160.0000	170.0000
c3	50.0000	170.0000	225.0000	+infinity
c4	50.0000	160.0000	180.0000	+infinity
c5	-5.0000	150.0000	160.0000	215.0000
c6	-10.0000	150.0000	160.0000	215.0000
c7	-15.0000	zero	160.0000	215.0000
c8	-15.0000	zero	160.0000	180.0000

In the table above the first four constraints (c1,c2,c3,c4) correspond to the constraints for satisfying demand during years 2000 to 2003, respectively. The last four constraints (c5,c6,c7,c8) correspond to the capacity constraints at the Morisoli vineyard during years 2000 to 2003, respectively.

(b) The V. Sattui winery is considering some preventive maintenance during one of the first 3 years. If maintenance is scheduled during 2000, it can only produce 151 units of Cabernet during this year (instead of 160). Similarly, if maintenance is scheduled for 2001, it will be able to produce 153 units, and if it is scheduled for 2002, the production will be 155 units during that year. When do you recommend that the winery schedules the maintenance? Why?

(c) Another winery has offered to supply up to 50 units of Cabernet during either 2000, 2001, or 2002 at a price of \$45 per unit. Should the V. Sattui winery buy from this other winery? If yes, when and how many units? What is the impact of this decision on the total cost?

(d) The provider of the Italian Barolo wine has offered to lower the price (from \$50 per unit) during 2001. What is the minimum decrease that would make this offer attractive to the V. Sattui winery?

(e) Because of anticipated inflation and increases in interest rates, the inventory holding cost is expected to increase to \$8 per unit during 2001. How does this affect the total cost and the optimal solution?

Solution

(a)

$$
\begin{array}{rl}
\text{minimize} & 35\sum_{i=1}^{4} x_i + 50\sum_{i=1}^{4} s_i + 5\sum_{i=1}^{4} I_i \\
\text{subject to} & \\
c1: & x_1 + s_1 - I_1 = 150, \\
c2: & x_2 + s_2 + I_1 - I_2 = 160, \\
c3: & x_3 + s_3 + I_2 - I_3 = 225, \\
c4: & x_4 + s_4 + I_3 - I_4 = 180, \\
c5: & x_1 \le 160, \\
c6: & x_2 \le 160, \\
c7: & x_3 \le 160, \\
c8: & x_4 \le 160, \\
 & x_i, s_i, I_i \ge 0, \qquad \forall i.
\end{array}
$$

(b) We consider constraints c5,c6, and c7 and the possibility of reducing the rhs (by 9, 7, and 5, respectively) to 151, 153, and 155 respectively. These values are within the range of allowable decrease given in the statement of the problem, which implies that the optimal solution does not change in either case. The rate of the increase in the cost is given by the corresponding dual variable ($-5, -10$, and -15 respectively). In particular, if maintenance is scheduled for 2000 the increase in the cost is $-9 \times -5 = 45$, if it is scheduled for 2001 the increase in the cost is $-7 \times -10 = 70$, and if it is scheduled for 2002 the the increase in the cost is $-5 \times -15 = 75$. So it is preferable to schedule the maintenance during 2000.

(c) By looking at the optimal solution, it makes sense to buy only during 2003. In particular, V. Sattui can buy 50 units and reduce s_3^* to 5 units, realizing savings of $\$5 \times 50 = \$250$ since there is \$5 difference between the Italian Barolo and the Cabernet from this other winery.

(d) Note that $s_2^* = 0$, so V. Sattui is not using Barolo during 2001. To increase s_2^* from zero the cost is \$5 per unit (the reduced cost of variable s_2). So the price of the Barolo should be decreased by at least \$5 per unit to make it attractive during 2001.

(e) The cost of I_2 can be increased up to \$10 without affecting the optimal solution. Thus, the increase to \$8 is within this range. The optimal solution will not change but the cost will increase by $\$3 \times 10 = \$30$ since $I_2^* = 10$.