# Problem Set 10

**Due: Tuesday, December 9, 2014**

**Readings**: The parts of chapters 10 and 11 from [BT] that we covered in class.

**Problem 1**

Consider the problem of scheduling $n$ jobs on $m$ machines that can work in parallel. Job $j$ requires processing time $p_{ij}$ on machine $i$. Assume that we start processing the jobs at time zero and let $C_j$ be the completion time of job $j$. The objective is to find a schedule that minimizes $\sum_{j=1}^{n} C_j$. Formulate the problem as an integer programming problem.

**Problem 2**

Consider the following integer programming (IP) problem

$$\begin{aligned} \text{maximize} \quad & \mathbf{c}'\mathbf{x} \\ \text{subject to} \quad & \mathbf{A}\mathbf{x} \leq \mathbf{b} \\ & \mathbf{x} \geq \mathbf{0} \\ & \mathbf{x} : \text{integer}, \end{aligned}$$

where $\mathbf{A}, \mathbf{b}$ and $\mathbf{c}$ are all composed of *positive* integers. Let $\mathbf{x}_0$ its optimal solution. Consider now the LP relaxation and let $\mathbf{x}_1$ its optimal solution.

**(a)** Show that $\lfloor \mathbf{x}_1 \rfloor$ is a feasible solution of IP.

**(b)** Show that the cost of $\lfloor \mathbf{x}_1 \rfloor$ can be no further from the optimal IP cost than $\sum_{i=1}^{n} c_i$.