Lecture 9: Outline

1. Feasible direction methods.
2. Conditional gradient method.
3. Gradient projection methods.
4. Two metric projection methods.
5. Manifold suboptimization - Quadratic programming.
Feasible direction methods

\[ x^{k+1} = x^k + \alpha^k d^k \]

such that \( d^k \) is feasible descent direction

- feasible direction: \( x^k + \alpha d^k \in \mathcal{X} \) ∀ small enough \( \alpha > 0 \).
- descent direction: \( \nabla f(x^k)'d^k < 0 \) if \( x^k \) is not stationary.
- Since \( \mathcal{X} \) is convex feasible directions:

\[ d^k = \gamma(\bar{x}^k - x^k), \quad \text{for } \gamma > 0 \text{ where } \bar{x}^k \in \mathcal{X} \text{ and } \bar{x}^k \neq x^k. \]

Conditional gradient method

\[ x^{k+1} = x^k + \alpha^k d^k \]

such that \( d^k \) is the feasible descent direction generated as follows:

\[ d^k = \bar{x}^k - x^k, \]

where \( \bar{x}^k \) is an optimal solution of

\[ \min \nabla f(x^k)'(x - x^k) \]

s.t. \( x \in \mathcal{X} \).
Gradient projection method

\[ x^{k+1} = x^k + \alpha^k d^k \]

such that \( d^k \) is the feasible descent direction generated as follows:

\[ d^k = \bar{x}^k - x^k, \]

where

\[ \bar{x}^k = [x^k - s^k \nabla f (x^k)]^+ \]

Two-metric projection method:

\[ \bar{x}^k = [x^k - s^k D^k \nabla f (x^k)]^+ \]